complete investigation of the question these residuals form the best possible material. When, therefore, the reductions for the Astrographic Catalogue are completed it will be possible to discuss very accurately the personal equations depending on magnitude in the observations for the meridian catalogues upon which it is based.

In conclusion, I must express my thanks to Professor Turner for the results from the Oxford University Observatory which he has kindly placed at my disposal, and I am also indebted to him and to Mr. Newall for much very helpful advice and criticism.

The Observatory, Cambridge:
1897 *April* 8.

An Investigation concerning the Position Error affecting Eye-Estimates of Star Magnitudes. By Alexander W. Roberts.

Measures of star magnitudes according to the method of Argelander, or any of its modifications, are always liable to be seriously vitiated by several disturbing influences. Attention has been directed from time to time to these sources of error by astronomers who have made special study in this branch of astronomical research; not infrequently they have been pointed out in order to throw discredit on the determination of star magnitudes by eye-estimates.

It is evident, however, that the mere existence of error in any series of measurements, of whatever nature, cannot condemn the measurements made, unless it is also certain that the errors cannot be avoided or eliminated. Now with ordinary precautions the chief errors that influence eye-estimates of stellar brightness can be entirely negatived, either by avoiding them or by adopting expedients which render them inoperative.

One of the principal influences modifying almost every observation made by eye-estimates is that arising from the ever-changing relative position of a variable and its surrounding comparison stars. Put generally, the error arises from the tendency, common, apparently, to all observers, to over-estimate the brightness of the lower star or stars in any field. If the amount of error were insignificant, it might be neglected without endangering seriously the substantial accuracy of any determination depending upon light measurements; but the amplitude of error is frequently almost a whole magnitude, an amount much greater than the actual variation of some shorter-period variables. Then in addition to the magnitude of error arising from this source we have its systematic nature. Now it is evident that a subjective systematic variation in the magnitude of any star whose light changes are under consideration will find expression

N N 2

in definite terms, the derivation of which it will be impossible to assign either to subjective change due to unequal sensitiveness of different parts of the retina, or to actual objective changes in the light of the star itself.

Thus we may have an apparent certain variation in the light of a star that after all is but the result of this tendency to over-estimate the brightness of the lower of two objects. It is pertinent, therefore, to the subject of stellar astronomy to investigate as thoroughly as possible a phenomenon that bears so directly on the accurate determination of star magnitudes.

The error may be avoided by taking observations when the star is in the same hour angle. This is not, however, always possible, and with rapidly varying stars it is out of the question. Then, again, the error may be avoided by the observer keeping his head always in the same position with relation to the stars under examination. For very evident reasons this expedient has been found to be practically unworkable. Another method of overcoming the difficulty will naturally suggest itself to many. The field under examination might always be kept in the same position with respect to the vertical by means of a rotating prism. In this case "position error" (using this term to describe the error now under consideration) would affect the absolute magnitudes of each star, but the excess or defect being always a constant quantity, the variation of the star would in no way be influenced.

None of the foregoing methods yield any data for an investigation as to the amount and nature of "position error." They are, indeed, effective expedients for its elimination, but they afford no means of ascertaining the amount of error eliminated.

I have had it in purpose for some years to adopt only such a mode of observation as would yield simply and readily the amount of error due to subjective causes, and the method that seemed best suited for such a purpose was that in which two observations are always taken—one direct, and the other reverse. This is secured by using for each observation two eye-pieces—one astronomical, and the other terrestrial.

It is evident that whatever stars are over-estimated when using one eye-piece, will be under-estimated to an equal extent when the other eye-piece is used. Thus a mean of the two observations will yield a set of measures entirely freed from position error. But, further, the difference between the two sets of measures (when properly reduced to a common standard) will yield *twice the amount of position error affecting any one of the stars at the time of observation.* Then if complete observations of the same set of stars be made in at least twelve hour angles, we shall have all the data necessary for a full investigation of the relation existing between the varying magnitudes and the changes in position producing the variation.

Instead of utilising all available observations I think it will give definiteness to the investigation to limit the area under

examination to one group of stars. For such an investigation several groups of stars are available, but I think the group that yields itself most readily to the investigation is that surrounding the short period variable, Lac. 5861.

The positions of this variable and its surrounding comparison stars are as follows:—

A.Z.C.	Lac.	R.A. (1875). h m s	Dec. ° ′ ″	Mag.
259	5834	14 3 57·6	−57 43 39	7·61
505	5859	7 48·1	56 42 45	7·26
512	...	7 51·1	57 44 29	7·87
533	5861	8 10·2	57 16 12	Var.
639	5870	9 56·8	58 6 21	7·40
837	5888	14 13 8·0	57 42 58	7·56

It will be evident from the accompanying chart that the configuration of the group (fig. 1) is in every way advantageous

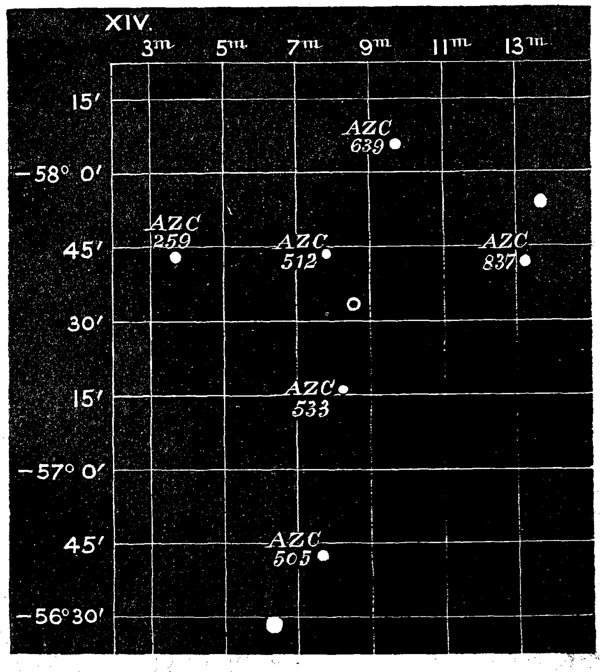


Fig. 1.

to a full investigation. Attention at this point may be called to the position marked O in the chart; this is the centre of the field, and is found in the usual way by triangulating the stars in the system.

The coordinates of O are:—

R.A. 14h 8m·5
Dec. −57° 33′·0.

The line passing from the zenith through this point will be called the vertical line.

The mode of reducing the estimated magnitudes to standard values bears directly on the question under consideration, and may briefly be described here.

Let

$$M_{(1)} + M_{(2)} + M_{(3)} + M_{(4)} + M_{(5)} = N$$

be the magnitudes of the five comparison stars A.Z.C. 259, 505, 512, 639, 837, as determined by sequences from $6^{m}\cdot8$ at a single observation, N being the sum total of the five magnitudes.

Let M_v be the magnitude of the variable A.Z.C. 533 (Lac. 5861) as determined with relation to the five comparison stars.

From all the observations a mean value of N is found: the average of over 500 values of N gave as this mean value

$$37\cdot70.$$

Then to reduce the magnitudes of the five comparison stars to mean standard value, the correction

$$\frac{37\cdot70 - N}{5}$$

is applied to each star, making the sum total of the five magnitudes,

$$M_{(1)}, M_{(2)}, M_3, M_{(4)}, M_{(5)},$$

always equal to $37\cdot70$.

The same correction is also applied to the variable, which therefore becomes

$$M_v + \frac{37\cdot70 - N}{5}.$$

This reduction is rigorously carried out for all observations direct and reverse.

The reduction being completed, the difference between the reverse and direct observations will represent twice the error due to the relative positions of the six stars in the group, the variable and the five comparison stars.

In the accompanying table the differences are taken in groups of one hour, and the parallactic angle of O is given in the same table, as the reduction of the differences will naturally be related in the first instance to this angle.

TABLE I.

Rot. No.	No. of Obs.	Sidereal Time.	Parallactic Angle.*	Reverse—Direct Observations. A.Z.C. 259.	A.Z.C. 505.	A.Z.C. 512.	A.Z.C. 533.	A.Z.C. 639.	A.Z.C. 837.
		h m	° ′	m					
1	4	5 46	44 12	−0·54	+0·54	−0·14	+0·16	−0·44	+0·46
2	4	6 37	54 0	·62	·53	·19	·18	·47	·55
3	5	7 29	63 50	·57	·53	·21	·13	·47	·57
4	16	9 40	89 39	·79	·44	·18	·13	·21	·62
5	26	10 27	100 11	·82	·30	·17	+0·09	−0·05	·65
6	28	11 27	115 46	·83	+0·14	·11	−0·02	+0·04	·77
7	35	12 32	137 52	·71	−0·32	−0·02	·09	·29	·85
8	49	13 29	161 1	·46	·57	0·03	·22	·57	·65
9	61	14 27	188 42	−0·19	·65	·03	·32	·73	·38
10	55	15 29	216 14	+0·08	·68	·04	·33	·78	+0·11
11	32	16 29	237 39	·39	·73	·10	·23	·68	−0·20
12	37	17 33	255 27	·61	·63	·13	·11	·51	·51
13	40	18 29	268 24	·73	·54	·20	·10	·34	·65
14	42	19 26	280 10	·80	·34	·18	·07	+0·16	·74
15	38	20 24	291 26	·87	−0·06	·13	·04	−0·05	·85
16	20	21 27	303 20	·78	+0·20	·05	−0·00	·11	·95
17	8	22 32	315 48	+0·67	+0·49	+0·02	+0·10	−0·32	−0·93

If the residuals in the preceding table be plotted down, and a curve drawn through the positions, it will be found that all the five curves can be well represented by an expression of the form,

$$x + \alpha \cos (q - M) + \beta \cos (2q - N).$$

From the nature of the error under investigation the first term of the above expression should be sufficient to satisfy the residuals of Table I., but as it is extremely improbable that observations 180° of parallactic angle, apart, will be taken under the same conditions, the second term is retained.

A rigorous least-squares solution of the seventeen sets of observations given in Table I. (weighting each set according to the number of observations in the set) yielded the following numerical values for the variation of the position error, depending on q, the parallactic angle at O.

(1) A.Z.C. 259.

$$-0^{m}\cdot04 + 0^{m}\,82 \cos (q - 288^{\circ}\ 22') + 0^{m}\cdot08 \cos (2\,q - 270^{\circ}).$$

* The *parallactic angle* is here considered to be the angle formed at O by the lines passing through the zenith and the south pole.

(2) A.Z.C. 505.

$$+0^{m}\cdot04+0^{m}\cdot85\cos(q-27^{\circ}\ 34')+0^{m}\cdot08\cos(2\,q-357^{\circ}).$$

(3) A.Z.C. 512.

$$-0^{m}\cdot04+0^{m}\cdot18\cos(q-240^{\circ}\ 50')+0^{m}\cdot06\cos(2\,q-236^{\circ}).$$

(4) A.Z.C. 533.

$$-0^{m}\cdot05+0^{m}\cdot22\cos(q-28^{\circ}\ 40')+0^{m}\cdot06\cos(2\,q-198^{\circ}).$$

(5) A.Z.C. 639.

$$+0^{m}\cdot05+0^{m}\cdot70\cos(q-200^{\circ}\ 50')+0^{m}\cdot07\cos(2\,q-97^{\circ}).$$

(6) A.Z.C. 837.

$$+0^{m}\cdot05+0^{m}\cdot82\cos(q-121^{\circ}\ 47')+0^{m}\cdot09\cos(2\,q-72^{\circ}).$$

A comparison between the observed values given in Table I. and the values computed from the preceding expressions is given in the following table.

Rot. No.	A.Z.C. 259. Obs. m	Comp. m	O−C. m	A.Z.C. 505. Obs. m	Comp. m	O−C. m	A.Z.C. 512. Obs. m	Comp. m	O−C. m	A.Z.C. 533. Obs. m	Comp. m	O−C. m	A.Z.C. 639. Obs. m	Comp. m	O−C. m	A.Z.C. 837. Obs. m	Comp. m	O−C. m	Rot. No.
1	−0·54	−0·49	−0·05	+0·54	+0·85	−0·31	−0·14	−0·26	+0·12	+0·16	+0·15	+0·01	−0·44	−0·52	+0·08	+0·46	+0·31	+0·15	1
2	·62	·60	−0·02	·53	·77	−0·24	·19	·25	+0·06	·18	·16	+0·02	·47	·47	+0·00	·55	·44	+0·11	2
3	·57	·70	+0·13	·53	·67	−0·14	·21	·24	+0·03	·13	·15	−0·02	·47	·40	−0·07	·57	·54	+0·03	3
4	·79	·82	+0·03	·44	·43	+0·01	·18	·18	+0·00	·13	·12	+0·01	·21	·25	+0·04	·62	·72	−0·10	4
5	·82	·82	+0·00	·30	+0·22	+0·08	·17	·14	−0·03	+0·09	·08	+0·01	−0·05	−0·09	+0·04	·65	·74	−0·09	5
6	·83	·79	−0·04	+0·14	−0·03	+0·17	·11	·09	−0·02	−0·02	+0·02	−0·04	+0·04	+0·06	−0·02	·77	·76	+0·01	6
7	·71	·67	−0·04	−0·32	·24	−0·08	−0·02	−0·03	+0·01	·09	−0·11	+0·02	·29	·30	−0·01	·85	·74	+0·11	7
8	·46	·49	+0·03	·57	·47	−0·10	+0·03	+0·00	+0·03	·22	·23	+0·01	·57	·54	+0·03	·65	·65	+0·00	8
9	−0·19	−0·21	+0·02	·65	·68	+0·03	·03	·03	+0·00	·32	·32	+0·00	·73	·75	−0·02	·38	·43	−0·05	9
10	+0·08	+0·12	−0·04	·68	·77	+0·09	·04	·07	−0·03	·33	·30	−0·03	·78	·79	−0·01	+0·11	+0·01	+0·10	10
11	·39	·39	+0·00	·73	·72	−0·01	·10	·11	−0·01	·23	·24	+0·01	·68	·68	+0·00	−0·20	−0·23	+0·03	11
12	·61	·60	+0·01	·63	·59	−0·04	·13	·14	−0·01	·11	·16	+0·05	·51	·49	+0·02	·51	·50	−0·01	12
13	·73	·72	+0·01	·54	·45	−0·09	·20	·15	+0·05	·10	·10	+0·00	·34	·33	+0·01	·65	·67	+0·02	13
14	·80	·79	+0·01	·34	·28	−0·04	·18	·14	+0·04	·07	·06	−0·01	+0·16	+0·16	+0·00	·74	·77	+0·03	14
15	·87	·83	+0·04	−0·06	−0·10	+0·04	·13	·13	+0·00	·04	−0·02	−0·02	−0·05	−0·10	+0·05	·85	·84	−0·01	15
16	·78	·83	−0·05	+0·20	+0·10	+0·10	·05	·10	−0·05	−0·00	+0·01	−0·01	·11	·26	+0·15	·93	·87	−0·06	16
17	+0·67	+0·76	−0·09	+0·49	+0·31	+0·18	+0·02	+0·05	−0·03	+0·10	+0·04	+0·06	−0·32	−0·41	+0·09	−0·93	−0·84	−0·09	17

So far we have considered the values of the errors, the terms expressing their amount and variation, as depending upon the parallactic angle of point O, the central point of the group of six stars. We now arrive at an important transference of argument.

The line passing from the zenith through O we have agreed to call the vertical line. Let, for brevity, the angle formed at O by this vertical line and a line joining O with any one of the five stars be called the position angle. Does the position error reach a maximum (or a minimum) for all stars in the group now under consideration at the same position angle?

It is possible to solve this question graphically, but in a first investigation it is preferable to proceed otherwise.

Let O be the origin of coordinates, and let O_1 be another point to which the measurements of position angle and parallactic angle are referred. Then taking

α and β coordinates of O_1;
α_1 and β_1 coordinates of star;
κ parallactic angle at O when position error is at a maximum;
θ constant position angle when position error is at a maximum;

each maximum of position error (obtained from Table I.) will yield equations of the form

$$(\beta_1 - \kappa\alpha_1)\,\theta + \kappa\,(\theta\alpha + \beta) + (\alpha - \theta\beta) = (\kappa\beta_1 + \alpha_1).$$

It is unnecessary to burden the present paper with the numerical steps leading to the following results:

$$\alpha = +1'\cdot5$$
$$\beta = -4'\cdot3$$
$$\theta = 32^\circ.$$

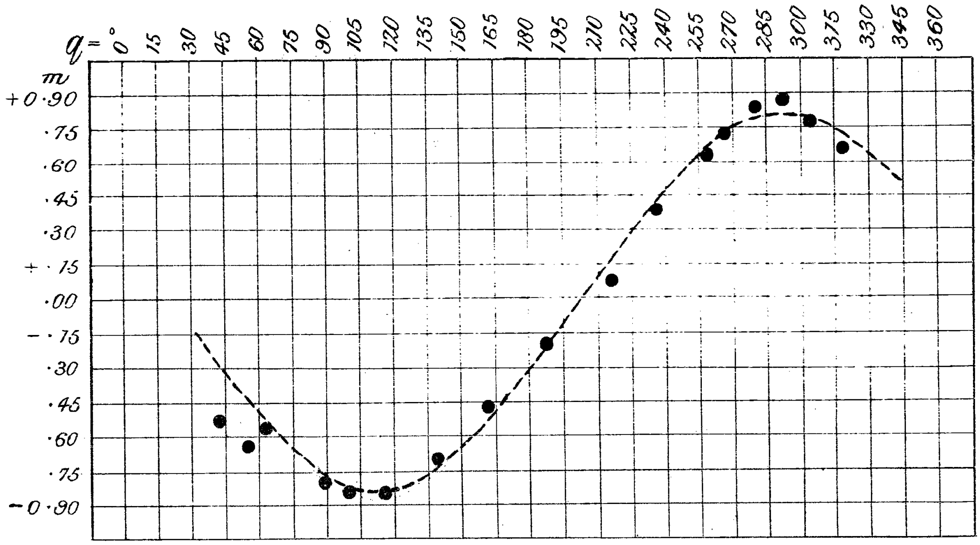
That the equation should yield as a mean point of reference a position only *four and a half minutes of arc* distant from the central point of the field is very remarkable when we consider the character of the investigation. Theoretically I am convinced that the two points O and O_1 coincide, but uncorrected sources of error—as, for example, a slight change in the position of the head when eye-pieces of different lengths are used—combine to produce the slight non-agreement of the two positions.

Taking, therefore, as our mean point of reference a position

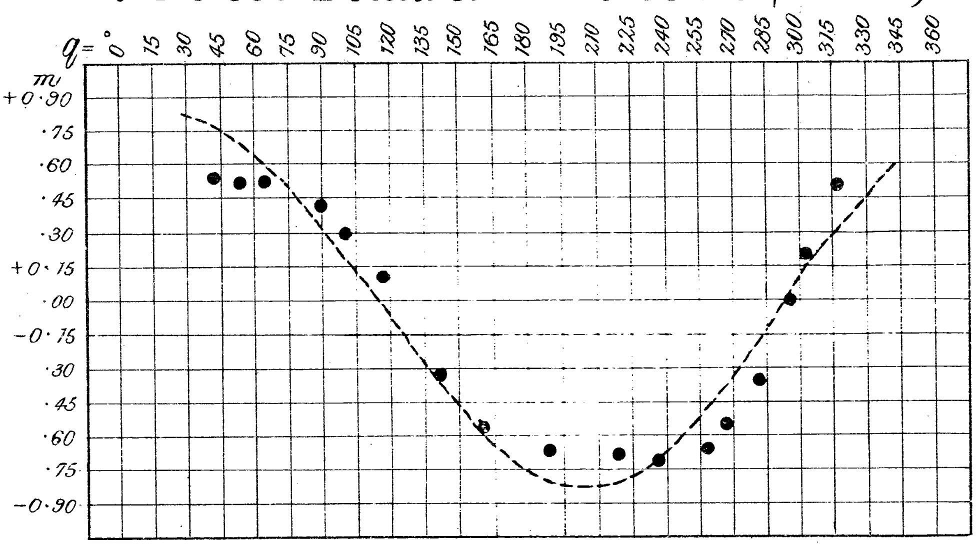
$$\text{R.A.}\quad 14^{\text{h}}\ 8^{\text{m}}\!\cdot7$$
$$\text{Dec.} - 57^\circ\ 37'\!\cdot3,$$

we find that the position errors affecting the magnitudes of the six stars under consideration reach their maximum when the position angle at O is 32°. Or, to put this statement more generally, with

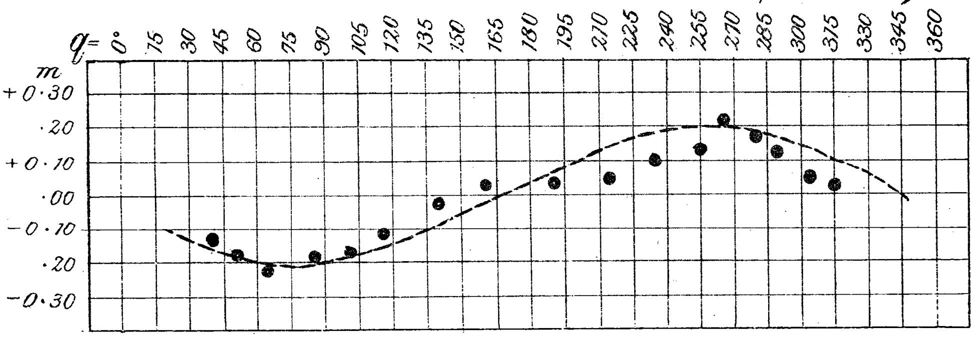
α Z.C. 259. Dotted Curve = $0^m.82 \cos(\theta - 32°)$



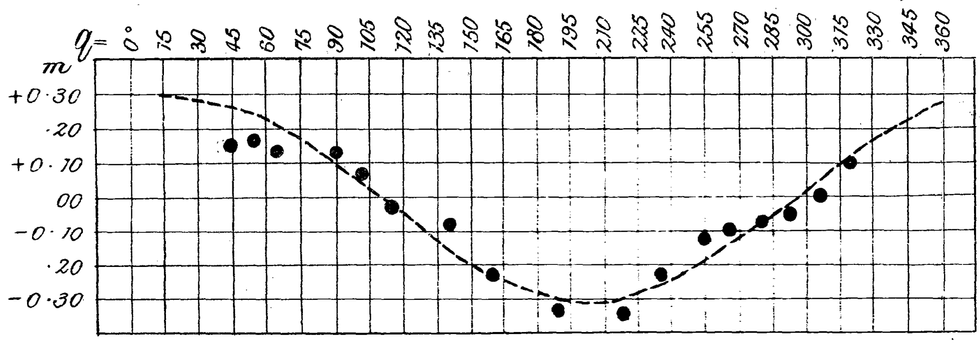
α Z.C. 505. Dotted Curve = $0^m.83 \cos(\theta - 32°)$



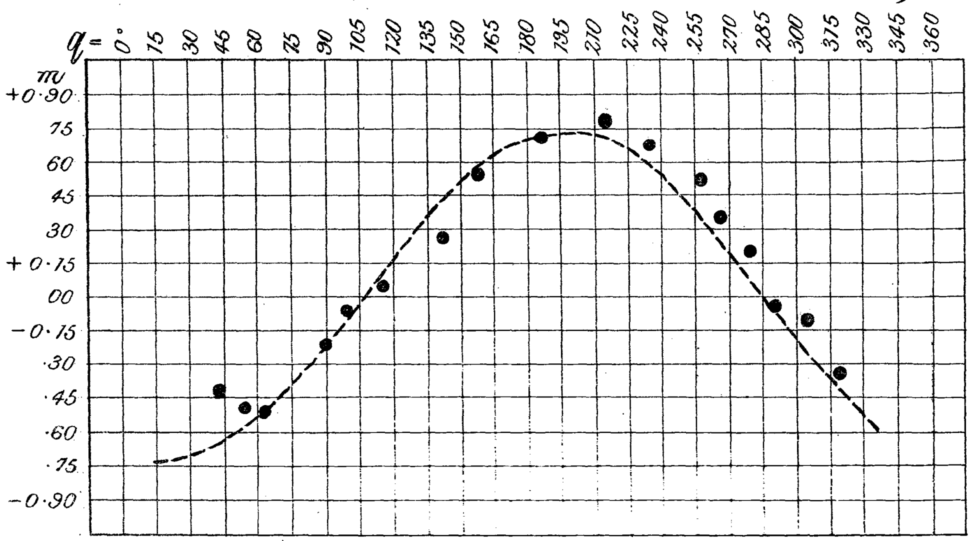
α Z.C. 512. Dotted Curve = $0^m.20 \cos(\theta - 32°)$



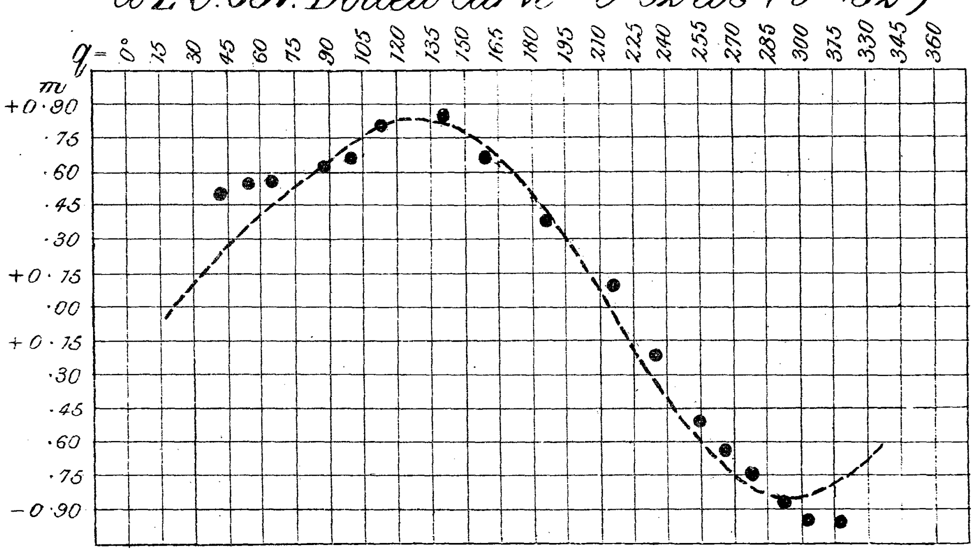
α Z C 533. Dotted Curve = $0^{m}{\cdot}30 \cos(\theta - 32^\circ)$



α Z C. 639. Dotted Curve = $0^{m}{\cdot}73 \cos(\theta - 32^\circ)$



α Z C. 837. Dotted Curve = $0^{m}{\cdot}82 \cos(\theta - 32^\circ)$



NOTE: θ = Angle formed at centre of field by the star & the nadir.

any system of stars whose magnitudes are determined visually each star will reach an apparent maximum brightness when the angle subtended by a line drawn from the star to the centre of the field and the vertical line at this point is 32°.

How close a general expression for all the six stars meets the observed differences between reverse and direct observation may be evidenced by graphical illustrations.

The dotted curves (Plates 12 and 13) are computed from the general formula

$$r \cos(\theta - 32^\circ).$$

I am unable to obtain a general expression for r, but it would seem that the further from the centre O the star is, the greater will the coefficient r be. The following table gives the relation :

	Star.	Distance of Star from O.	Observed Values of r.
		′	m
1	A.Z.C. 505	55·1	0·83
2	259	38·3	0·82
3	837	36·6	0·82
4	639	30·8	0·73
5	533	21·5	0·30
6	512	9·6	0·20

The disturbing effects of position error have been commented upon by Safarik (*Ast. Nach.* Band 101, page 24).

It will be seen that the relation existing between the distance of a star from the centre of the field and the amount of error which influences comparative eye-estimates of its brightness is by no means an intimate one ; indeed, we can readily understand circumstances—as, for example, twenty stars in a line—where the relation would naturally break down.

The foregoing investigation has been confined to the group of stars surrounding L. 5861 ; but whatever group be considered, as far as Lovedale observations are concerned, the general results are the same.

(1) The error due to relative position is systematic in its nature.

(2) It reaches its maximum effect when the position angle of the star with relation to the vertical at the centre of the field is 32°.

(3) It affects all magnitudes equally.

These results are absolute as far as my personality is concerned ; whether they hold good for other observers I cannot say. It must not be forgotten, however, that the error has been commented upon at some length by Safarik (*Ast. Nach.* Band 101, p. 24); by Pickering (*Harvard Annals*, vol. xiv. part 1, p. 42); and by Chandler (*Astronomical Journal*, No. 374, p. 107). The two latter astronomers have also proved in the papers referred to the systematic nature of the error. I may be permitted also to say

that Dr. Gould, when corresponding with me some years ago on the matter, informed me that the phenomenon was frequently an anxiety to the Cordoba observers. It is not unreasonable, therefore, to conclude that the error, in at least its general characteristics, influences all eye-estimates of star magnitudes by the method sequences. That there are possibilities, therefore, of grave misleading in discussing observations, in the securing of which no steps have been taken, either through want of care or want of knowledge, to eliminate so important an error, is surely beyond reasonable doubt. Indeed, it is certain that every result based on uncorrected observations is more or less in error.

Whether some of the secondary terms that appear so often in the computed elements of variable stars have their origin in this source of error I cannot say ; only it is possible for position error to give rise to such secondary terms when left uncorrected. It is unnecessary to dwell upon the connection existing between this tendency and the exceedingly long lists of suspected variables which now and again find a lodging place in astronomical literature.

The assumption all through this paper, as well as in the papers of Safarik, Pickering, Chandler, and Barnard, is that the error has its source in the eye of the observer. I have convinced myself that this is so by a very simple expedient. The telescope I use is a small 1-inch theodolite. To make sure that the error was not due to the nature or adjustments of the instrument, I rotated the small telescope in its Ys, thus taking the observations with the object glass in all positions. I also used a variety of eye-pieces. Further, when the head was rotated through a certain angle, the same changes in the relative magnitudes of the stars seemed to take place as would have been observed had the stars moved through an equal hour angle.

The error also operates in cases when no optical aid is required.

Proper Motion of the Southern Short Period Variables L Carinæ and K Pavonis. By Alexander W. Roberts.

The following proper motions of *L Carinæ* and *K Pavonis* have been determined solely from observations made at the Cape Observatory. The measures available are the positions given in the General Catalogues for 1840, 1850, 1860, 1880, and the meridian results of 1887, 1888, 1889, 1890, 1891.

The mean places are reduced to 1900·0, using Newcomb's precession constants (*Astron. Constants,* p. 196), and the mean latitude of the Cape Observatory is held to be

$$-33^\circ\ 56'\ 3''\cdot52.$$

Whenever possible Chandler's corrections for latitude variation have been applied, and the various catalogues have been differentially compared.